Students' Name:_____Teachers' Code: Please circle NHM JPN MXF FEY SJM

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2022

Year 12
Mathematics
Task 4 Trial HSC Exam
Date: Thursday 25th August 2022

General Instructions:

- **Reading time : 10 mins**
- **Time Allowed: 3 hours**

- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions.
- A NESA Reference Sheet is provided.
- Questions 1 to 10 are all multiple-choice questions worth 1 mark each and are to be answered on the multiple-choice answer sheet provided.
- Questions 11 to 35 are each worth 90 marks each and are to be answered on the examination paper.
- Each booklet and the multiple-choice answer sheet must have **your name** and **the initials of your class teacher** on the front cover.
- Marks may not be awarded for missing or carelessly arranged working.

Topics Examined:

All Preliminary and HSC Mathematics topics

SECTION 1
Questions 1 -10 **10 Marks**

SECTION 2
Question 11 -35 **90 Marks**

Total **100 Marks**

Teacher:

- Mr N Mushan NHM
- Mr J Newey JPN
- Dr M Furtado MXF
- Ms F Yates FEY
- Mr S Maher SJM

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SECTION 1 (10 marks)

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ←

(A) ⊗ (B) ● (C) ○ (D) ○

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1. The domain of $y = \log_e(x+2)$ is:

A. $x > 0$

B. $x > 1$

C. $x > -3$

D. $x > -2$

2. Compared with the graph of $y = \cos x$, the graph of $y = \cos\frac{x}{2}$ has:

A. half the amplitude but the same period

B. the same amplitude and half the same period

C. double the amplitude and the same period

D. the same amplitude and double the period

3. A set of data is normally distributed with a mean of 8.6 and a standard deviation of 0.7. The percentage of scores that lie between 8.6 and 9.3 is

A. 17.5%

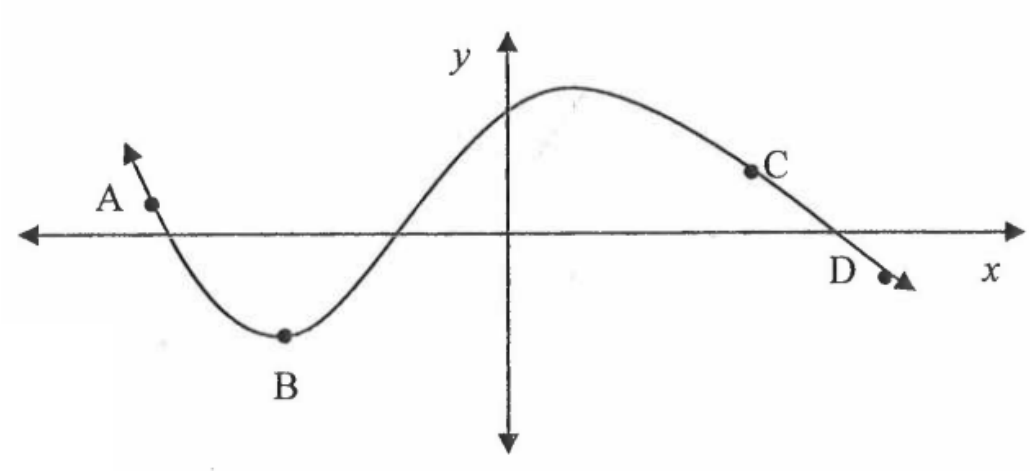
B. 34%

C. 68%

D. 95%

4. Which point on the following diagram relates to the following description.

$y > 0,\ \frac{dy}{dx} < 0,\ \frac{d^2y}{dx^2} < 0$



A. A

B. B

C. C

D. D

5. $\int_1^a 4-2x\ dx=-3$ where $a>0$. The value of a is

A. 1

B. 4

C. 6

D. 8

6. $\int_0^1 e^{3x}\ dx$

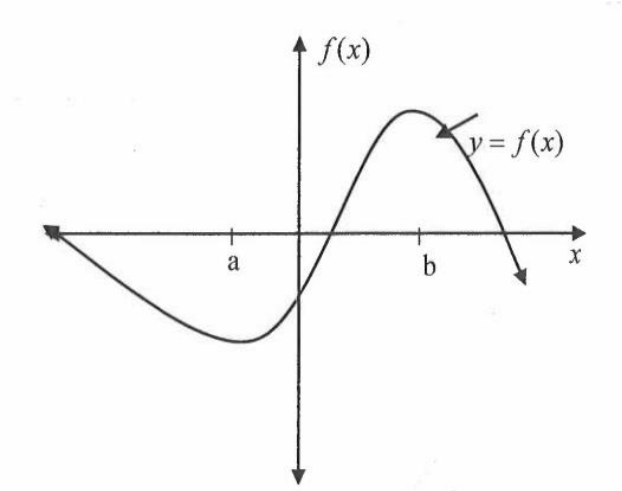
A. $1-e^3$

B. e^3-1

C. $\frac{e^3-1}{3}$

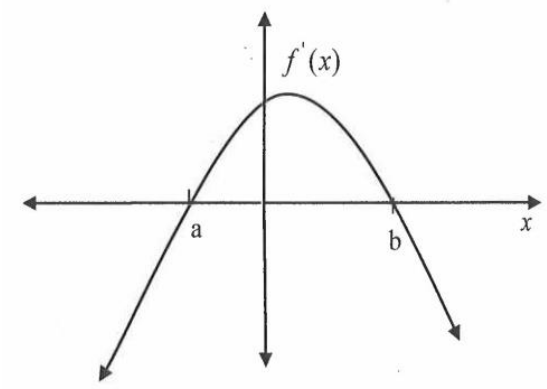
D. $\frac{1-e^3}{3}$

7. The diagram shows the graph of $y = f(x)$

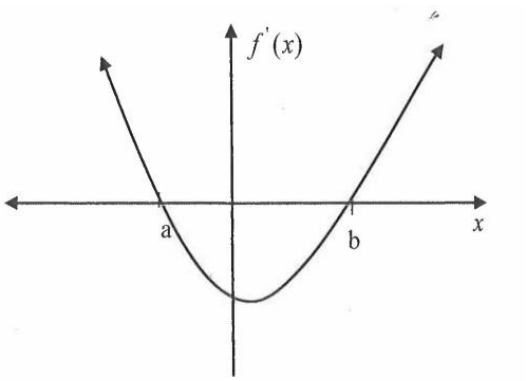


Which of the following is the graphs shows $y = f'(x)$?

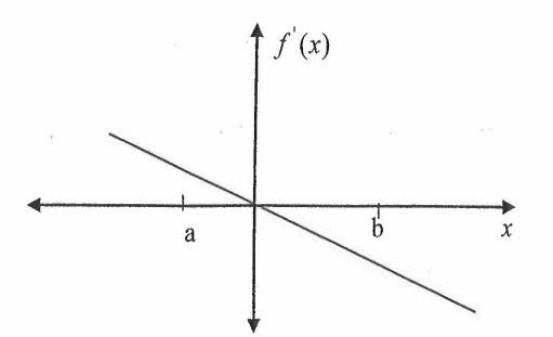
A.



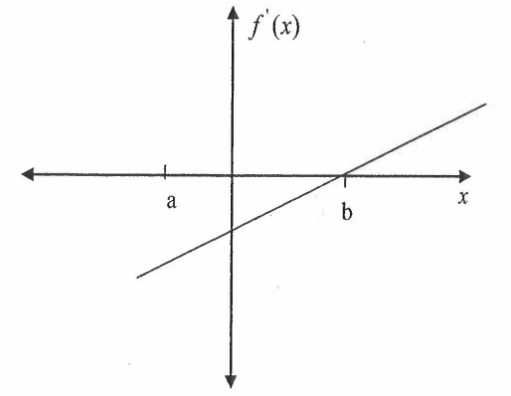
B.



C.



D.



8. The solutions to $e^{6x} - 5e^{3x} + 6 = 0$ are $x =$

A. $2, 3$

B. $\log_e 2, \log_e 3$

C. e^{2x}, e^{3x}

D. $\frac{1}{3}\log_e 3, \frac{1}{3}\log_e 2$

9. If $y = \log_a x$, then $\frac{dy}{dx} =$

A. $\frac{1}{x}$

B. $\frac{1}{x \log_e a}$

C. $\frac{1}{a}$

D. $\frac{1}{\log_e x}$

10. Given the curve $y = f(x)$, $\frac{d^2y}{dx^2} = (x+4)^2(x-2)$, which of the following statements are correct

A. $x = -4$ and $x = 2$ are both the x co-ordinates of points of inflection

B. $x = -4$ is the only x co-ordinate of a point of inflection

C. $x = 2$ is the only x co-ordinate of a point of inflection

D. $x = -4$ and $x = 2$ are both not the x co-ordinates of points of inflection

End of Section 1

SECTION 2

Total Marks – 90
Attempt Questions 11-31
[Marks for each part are indicated on the page]
Allow about 2 hours and 45 minutes for this section

Question 11 **Marks**

A rainwater tank which is full, is drained so that at time 't' minutes the volume of water V in litres is given by $V = 500\,(1-\frac{t}{60})^2$ for $0 \le t \le 60$.

(a) How much water was initially in the tank? **1**

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(b) After how many minutes was the tank half full? (correct to 2 decimal places) **2**

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(c) At what rate was the water draining when the time is 58 minutes. **2**

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Question 12

Find the sum of the series 8+14+20+26+ …+152. **2**

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Question 13

Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$. **2**

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Question 14

The ratio of boys to girls at Rivendale High School is 3:5. The ratio of boys to girls in South Park High school is 2:3. A student is chosen at random from Rivendale High School and then another student is chosen from South Park High School.

Find the probability that:

(a) two boys are chosen. **2**

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(b) at least one girl is chosen. **1**

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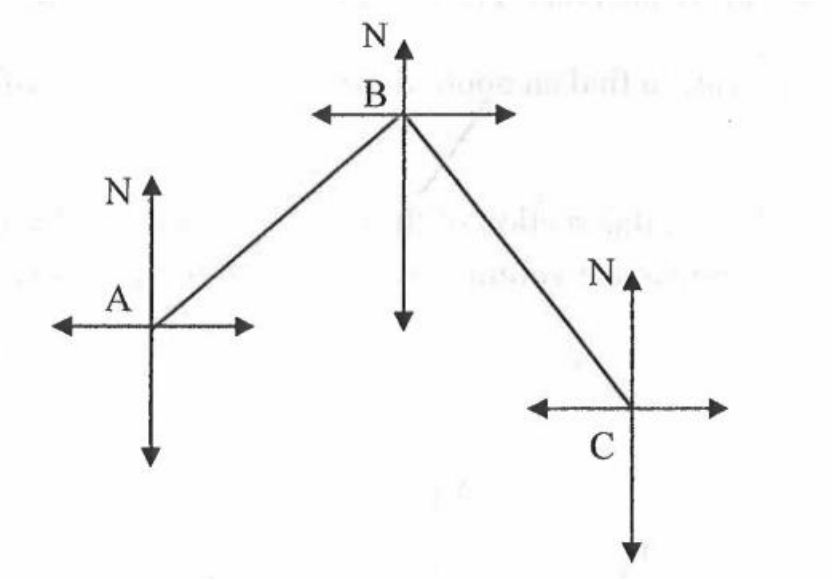
Question 15

A ship sails 150 km from Appleville (A) to Brooktown (B) on a bearing of 050° T.
It then sails on a bearing of 130° T to Cook (C) which is 300 km from Brooktown.



Label the diagram above showing all the essential features **1**

(a) Find the distance from Appleville (A) to Cook (C) **2**
(Give your answer correct to two decimal places)

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(b) What is the bearing of Appleville (A) from Cook (C)? **2**
(Give your answer to the nearest minute)

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Question 16

For the curve $y = \frac{x^3}{3} - 3x^2 + 8x + 5$

(a) Find the coordinates of the stationary points and distinguish their nature. **3**

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(b) Find the point of inflection. **2**

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(c) What is the maximum value of $y = \frac{x^3}{3} - 3x^2 + 8x + 5$ for the domain $x \in R\ [0, 6]$? **2**

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Question 17

(a) Evaluate $\int_2^3 \frac{x}{x^2-1}\,dx$. (Write your answer as an exact value) **2**

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(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$. (Write your answer as an exact value) **3**

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Question 18

Differentiate $\frac{x}{\tan 2x}$. **2**

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Question 19

Expand and simplify $(\cos\text{ec}\,A+1)(\cos\text{ec}\,A-1)$. **2**

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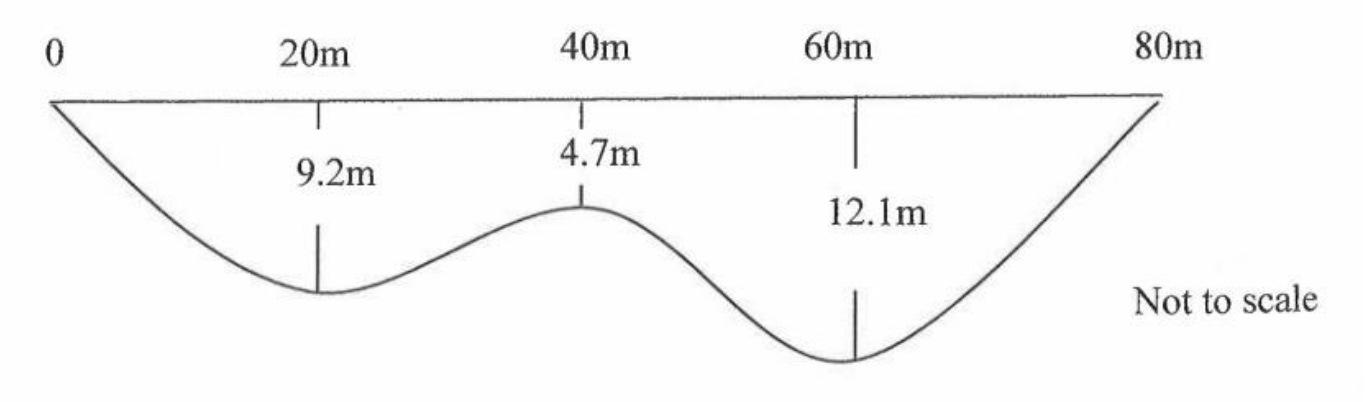
Question 20

The following diagram represents a cross-section through a river.
The depth of the river is marked every 20 metres.



Use the trapezoidal rule with 5 function values to estimate the area of the cross-section. **2**

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Question 21

(a) The temperature in the kitchen of Nameeta's new home is controlled by a thermostat.

The temperature can be modelled by the equation: $T = 21.7 + \sin\left(\frac{t}{2}\right)$ where

T is the temperature in degrees Celcius and t is the time in minutes since Nameeta entered the kitchen.

How long will Nameeta have to wait before the temperature gets to 22.5° C **2**

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(b) The volume V cm^3 of a balloon is increasing such that the volume at any time *t* seconds is given by $V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$.

Find the rate at which the volume is increasing when $t = 2$ **2**

(write your answer in exact form)

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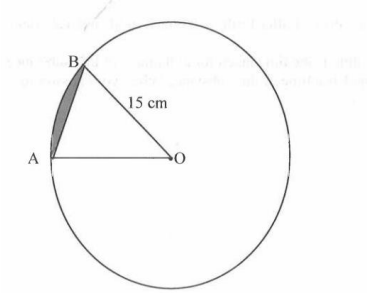
Question 22



The diagram is a circle centre O and radius 15 cm. The triangle AOB is an equilateral triangle.

AB is a chord of the circle.

(a) State in radians the size of angle AOB. **1**

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(b) Calculate the area of the shaded region. (Write your answer as an exact value) **3**

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Question 23

A uniform continuous random variable with probability density function is shown below.

$$f(x) = \begin{cases} ax^2, & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of a. **2**

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(b) Find $P(3 \le X \le 5)$ **3**

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Question 24

The circle $x^2 + 6x + y^2 - 4y - 3 = 0$ is reflected in the x-axis. **3**

Sketch **on the next page** the **reflected** circle, showing the coordinates of the centre and the radius.

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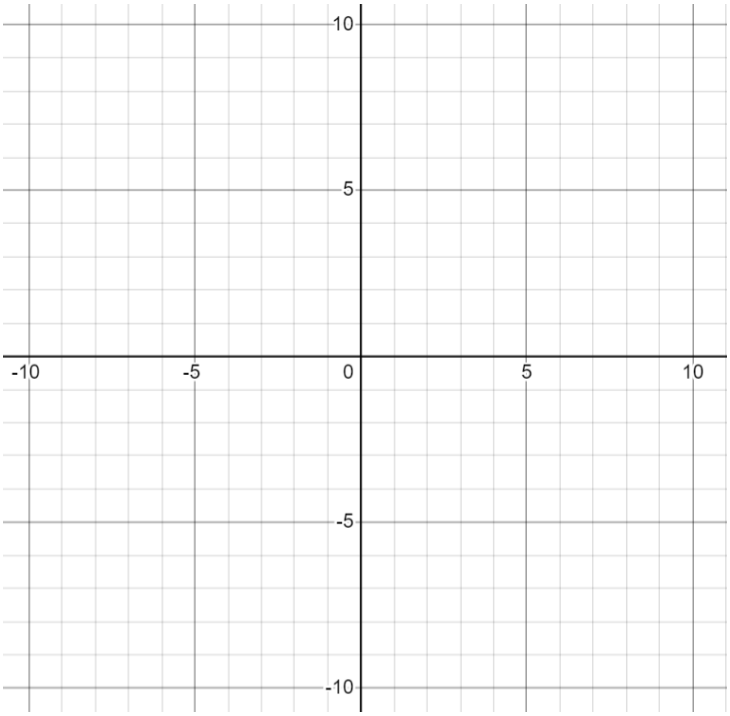

Question 25

Jason deposits an amount of $100 000 into an account which pays compound interest of 2% per annum, added to the account at the end of each year.
Immediately after the interest is added, Jason makes a withdrawal for expenses for the coming year.
The first withdrawal is $M. Each subsequent withdrawal is 10% greater than the previous one.

Let A_n be the amount of money in the account after the nth withdrawal.

(a) Show that $A_2 = 100\,000(1.02)^2 - M(1.02 + 1.10)$ **1**

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(b) Show that $A_3 = 100\,000(1.02)^3 - M[(1.02)^2 + (1.02)(1.10) + (1.10)^2]$ **1**

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(c) Hence, write an expression for A_n **1**

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(d) Show there will be NO money in the account when $M = \dfrac{8000}{\left[\left(\dfrac{1.10}{1.02}\right)^n - 1\right]}$ **2**

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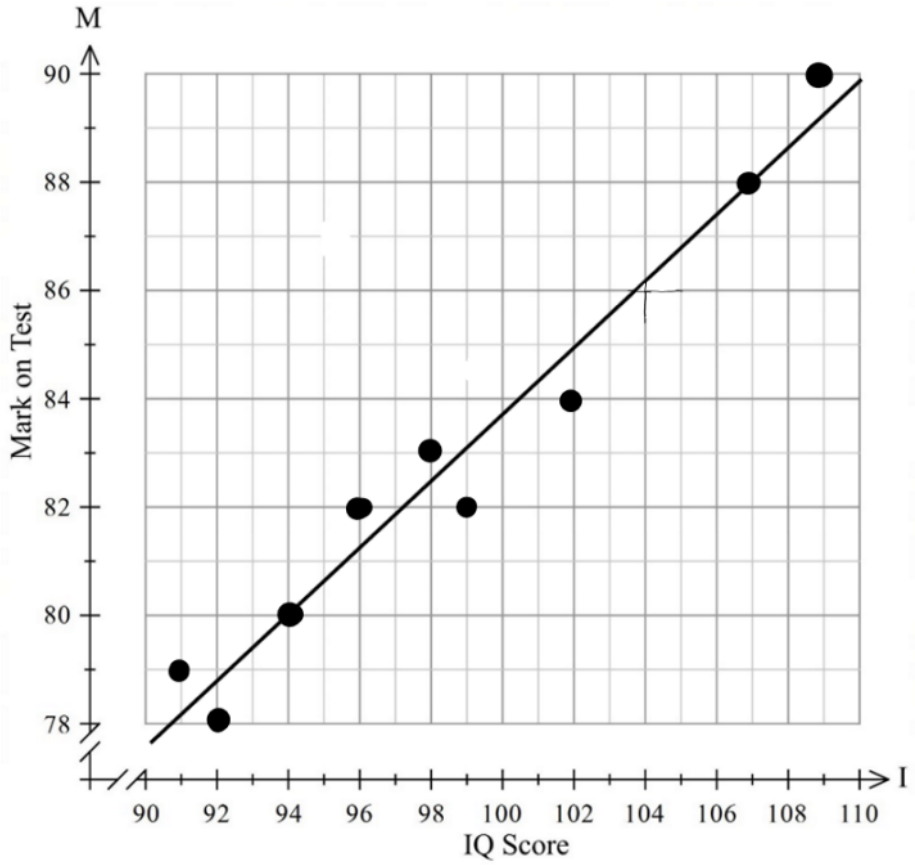
Question 26

The bivariate data in the scatterplot below compare the recorded IQ scores of students with their mark on a class test out of 100. A line of best fit was also drawn.



(Note: Both axes are truncated)

(a) Calculate the gradient of the line of best fit **1**

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(b) Describe the correlation between the IQ score and the mark on the test in terms of **direction** and **strength**. **2**

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Question 27

A continuous probability distribution is given by $f(x)=\frac{3x^2}{279}$ defined in the domain $[4,7]$

(a) Find the cumulative distribution function. **3**

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(b) Find the median of the continuous probability distribution. **2**

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Question 28

Evaluate $\int_1^3 x^2(x^3+8)^2\,dx$ **2**

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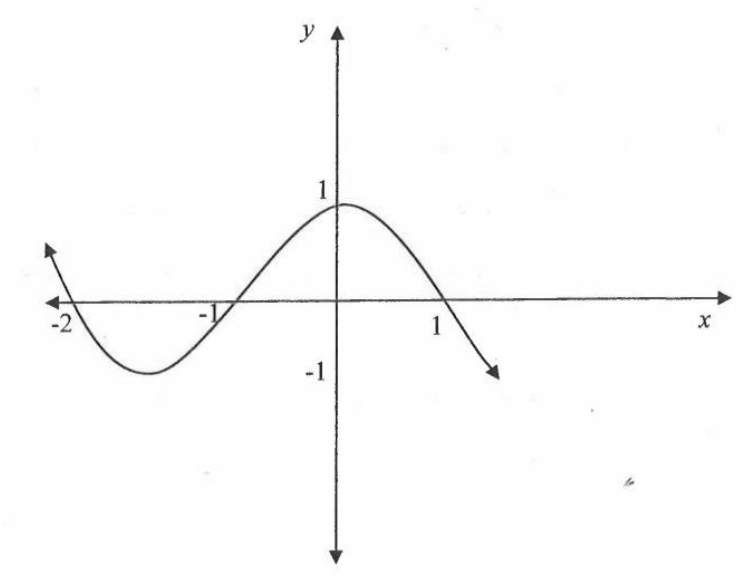
Question 29

(a) The graph of $y = f(x)$ is shown.



On the graph above draw the curve, $y = f(x-1)+1$, showing all important features. **2**

Question 30

A class of students sat a test worth 30 marks. The marks were normally distributed.

The mean was 18 and the standard deviation was 1.6.

(a) Calculate Julie's z-score if her mark was 22. **1**

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(b) If Dan's z-score was 1.25, calculate his actual mark. **1**

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Question 31

A psychologist proposes that the ability of a child to memorise during the first five years can be modelled by the function $f(x)=1+x\log_e x \ \ 0<x\leq 5$

(i.e the ability to memorise at age x years is $f(x)$)

(a) During which month is the ability at a minimum in the first five years **3**

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(b) When is it a maximum during this period. Give a reason for your answer. **2**

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Question 32

Evaluate $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y$, given that $y=xe^{-2x}$. **2**

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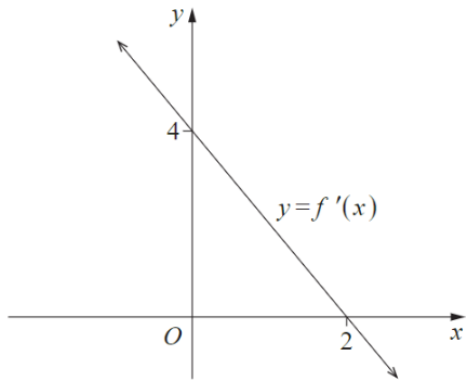
Question 33

The graph of the derivative of a function $y=f'(x)$ is shown below. **3**



The curve $y=f(x)$ has a maximum value of 12.

What is the equation of $y=f(x)$?

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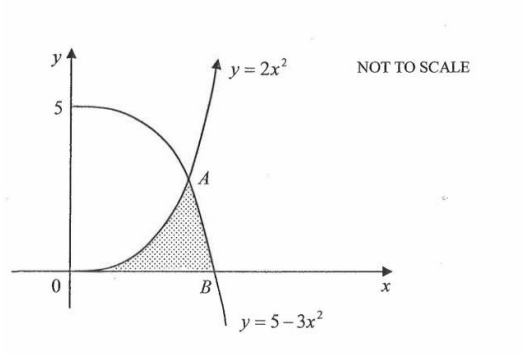
Question 34



The shaded region OAB is bounded by the parabolas $y = 2x^2$, $y = 5 - 3x^2$ and the x-axis.

Point A is the intersection of the two parabolas and point B is the x-intercept of the parabola $y = 5 - 3x^2$

(a) Show that the x- coordinates of A and B are 1 and $\frac{\sqrt{5}}{\sqrt{3}}$ respectively. **2**

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(b) Find the exact area of the shaded region OAB. **3**

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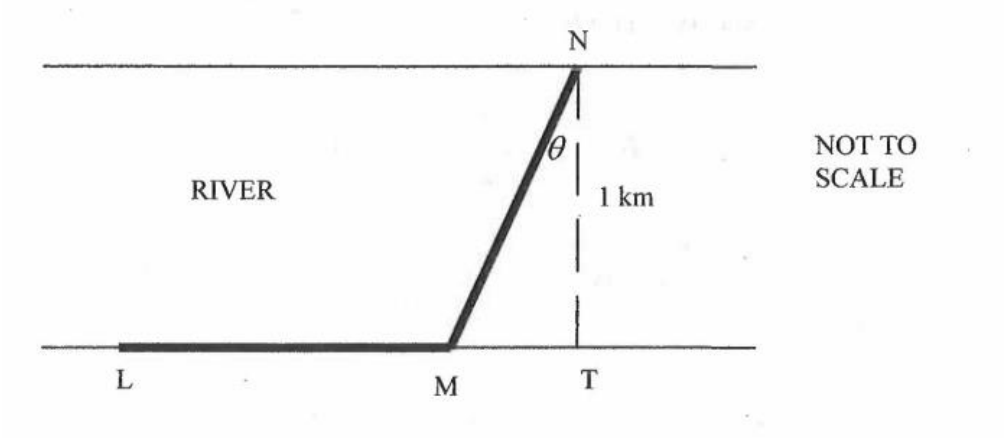
Question 35

It is desired to construct a cable link between two points L and N, which are situated on opposite banks of a river of width 1 km. L lies 3 km upstream from N. It costs 3 times as much to lay a length of cable underwater as it does to lay the same cable overland.

The following diagram is a sketch of cables where θ is the angle where NM makes with the direct route across the river.



Note: $MN = \sec\theta$ and $MT = \tan\theta$

(a) If segment LM costs c dollars per km, prove the total cost (T) of laying the cable is given by $T = 3c - c\tan\theta + 3c\sec\theta$ **2**

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(b) At what angle should the cable cross the river in order to minimise the total cost of laying it. **3**

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End of Exam

Students' Name:_____ Teachers' Code: Please circle NHM JPN MXF FEY SJM

SUGGESTED SOLUTIONS

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Total **100 Marks**

Teacher:

- Mr N Mushan Q32 – Q35 NHM
- Mr J Newey Q16 - Q21 JPN
- Dr M Furtado Q26 - Q31 MXF
- Ms F Yates Q22 - Q25 FEY
- Mr S Maher Q11 – Q15 SJM

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SECTION 1 (10 marks)

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ←

(A) ⊗ (B) ⊗ (C) ○ (D) ○

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1. The domain of $y = \log_e(x+2)$ is:

D: $x+2 > 0$

$x > -2$

A. $x > 0$

B. $x > 1$

C. $x > -3$

(D.) $x > -2$

2. Compared with the graph of $y = \cos x$, the graph of $y = \cos\frac{x}{2}$ has:

$y = \cos x$ period $= 2\pi$

$y = \cos\frac{x}{2}$ period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

A. half the amplitude but the same period

B. the same amplitude and half the same period

C. double the amplitude and the same period

(D.) the same amplitude and double the period

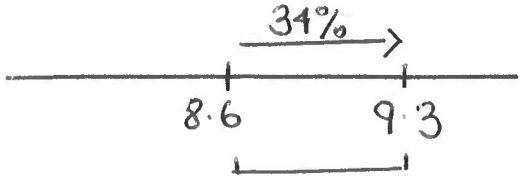
3. A set of data is normally distributed with a mean of 8.6 and a standard deviation of 0.7. The percentage of scores that lie between 8.6 and 9.3 is

A. 17.5%

B. 34%

C. 68%

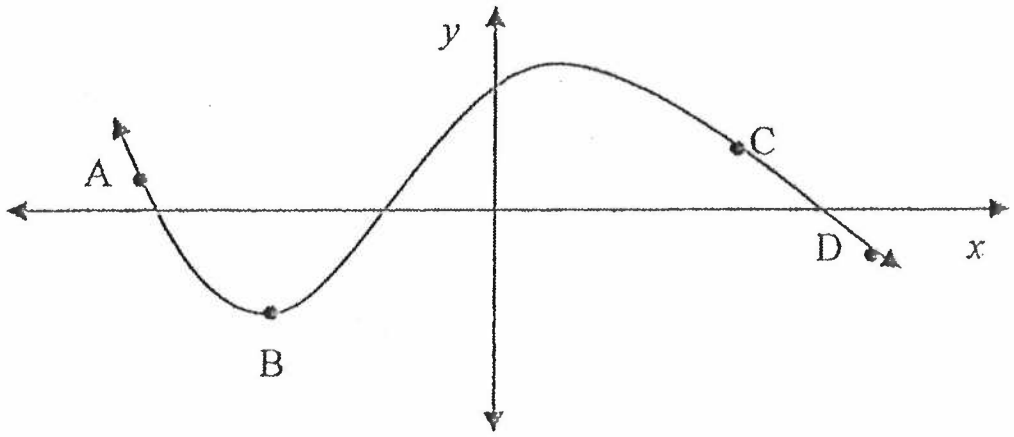
D. 95%



1 standard deviation 0·7

4. Which point on the following diagram relates to the following description.

$y > 0, \frac{dy}{dx} < 0, \frac{d^2y}{dx^2} < 0$



A. A

B. B

C. C

D. D

$y > 0$ A or C

$\frac{dy}{dx} < 0$ A or C (curve is decreasing)

$\frac{d^2y}{dx^2}$ (C) (concave down)

5. $\int_1^a 4-2x\ dx=-3$ where $a>0$. The value of a is

A. 1

B. 4

C. 6

D. 8

$$[4x-x^2]_1^a = -3$$

$$(4a-a^2)-(4-1) = -3$$

$$a^2-4a = 0$$

$$a = \emptyset \text{ or } 4$$

6. $\int_0^1 e^{3x}\ dx$

$$\frac{1}{3}[e^{3x}]_0^1 = \frac{1}{3}(e^3-1)$$

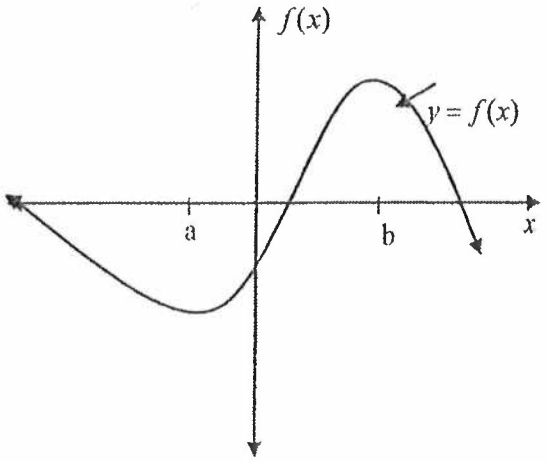
A. $1-e^3$

B. e^3-1

C. $\frac{e^3-1}{3}$

D. $\frac{1-e^3}{3}$

7. The diagram shows the graph of $y = f(x)$



original function: 'cubic'
derivative function: 'quadratic'
A or B

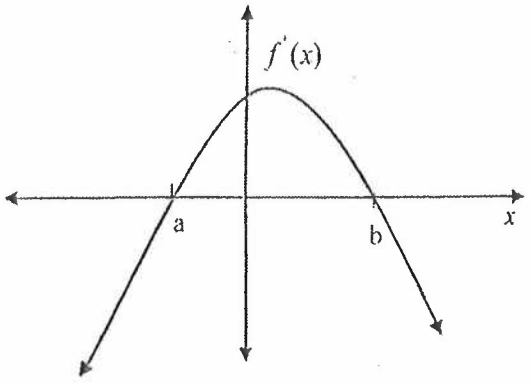
S.P at $x = a$ and b
derivative function 0 (at a and b)
A or B

Nature of SP
$x = a$ (min T.P) ∪
$x = b$ (max T.P) ∩

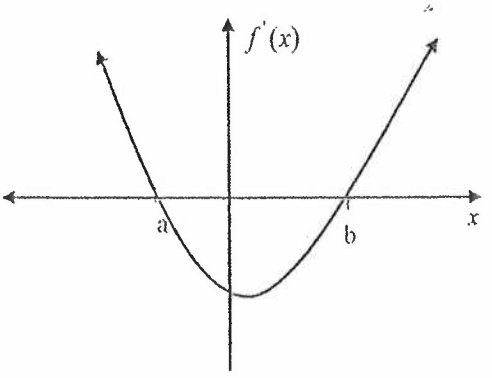
∴ (A)

Which of the following is the graphs shows $y = f'(x)$?

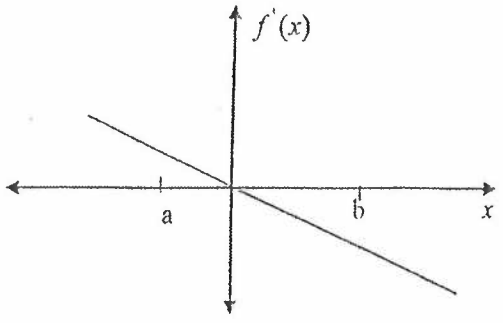
(A.)



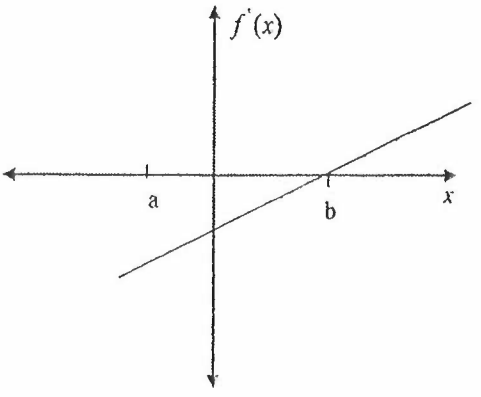
B.



C.



D.



8. The solutions to $e^{6x} - 5e^{3x} + 6 = 0$ are $x =$

$u^2 - 5u + 6 = 0$

$(u-2)(u-3) = 0$

$u = 2 \quad u = 3$

$e^{3x} = 2 \quad e^{3x} = 3$

$3x = \ln 2$ or $\ln 3$

A. $2, 3$

B. $\log_e 2, \log_e 3$

C. e^{2x}, e^{3x}

D. $\frac{1}{3}\log_e 3, \frac{1}{3}\log_e 2$

9. If $y = \log_a x$, then $\frac{dy}{dx} =$

$y = \frac{\ln x}{\ln a}$

$\frac{dy}{dx} = \frac{1}{\ln a} \cdot \frac{1}{x}$

A. $\frac{1}{x}$

B. $\frac{1}{x \log_e a}$

C. $\frac{1}{a}$

D. $\frac{1}{\log_e x}$

10. Given the curve $y = f(x)$, $\frac{d^2y}{dx^2} = (x+4)^2(x-2)$, which of the following statements are correct

$x = -5, \; y'' = -7$

$x = 0 \quad y'' = -32$

$x = 3 \quad y'' = 49$

A. $x = -4$ and $x = 2$ are both the x co-ordinates of a point of inflection

B. $x = -4$ is the only x co-ordinate of a point of inflection

C. $x = 2$ is the only x co-ordinate of a point of inflection

D. $x = -4$ and $x = 2$ are both not the x co-ordinates of a point of inflection

End of Section 1

SECTION 2

Total Marks – 90
Attempt Questions 11-31
[Marks for each part are indicated on the page]
Allow about 2 hours and 45 minutes for this section

Question 11 **Marks**

A rainwater tank which is full is drained so that a time 't' minutes the volume of water V in litres is given by $V = 500\,(1-\frac{t}{60})^2$ for $0 \le t \le 60$.

(a) How much water was initially in the tank? **1**

$V = 500$ L ✓ 1 mk

(b) After how many minutes was the tank half full? **2**

$(1-\frac{t}{60})^2 = \frac{1}{2}$ ✓ 1mk for progress

$t = 17.57$ (18 min) ✓ or $t = 102.4$ (reject) $0 \le t \le 60$.

1mk for correct solution

(c) At what rate was the water draining when the time is 58 minutes. **2**

$\frac{dV}{dt} = 1000(1-\frac{t}{60}) \times -\frac{1}{60}$ ✓ 1mk for derivative

when $t = 58$

$\frac{dV}{dt} = -5/9$ ✓ (draining at $5/9$ L/min

1mk for answer

Question 12

Find the sum of the series 8+14+20+26+ ...+152. **2**

$a+(n-1)d = 152$
$8+(n-1)6 = 152$
$n = 25$ ✓ 1mk for n

A.P $a = 8$ $d = 6$

$S_{25} = \frac{25}{2}[8+152]$

$= 2000$ ✓ 1mk for sum

Question 13

Evaluate $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$. 2

$\frac{1}{2}[\tan 2x]_0^{\pi/8}$ ✓ 1mk for integration

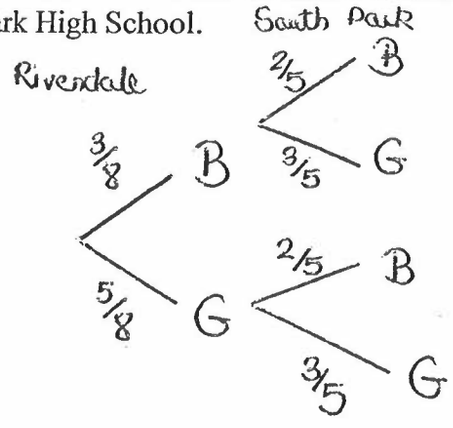
$= \frac{1}{2}[\tan \frac{\pi}{4} - \tan 0]$

$= \frac{1}{2}[1 - 0]$

$= \frac{1}{2}$ ✓ 1mk for substitution and answer

Question 14

The ratio of boys to girls at Rivendale High School is 3:5. The ratio of boys to girls in South Park High school is 2:3. A student is chosen at random from Rivendale High School and then another student is chosen from South Park High School.



1mk for method to determine $\frac{3}{8}$ and $\frac{2}{5}$

Find the probability that:

(a) two boys are chosen. 2

$P(BB) = \frac{3}{8} \times \frac{2}{5}$

$= \frac{3}{20}$ // 1mk for solution

(b) at least one girl is chosen. 1

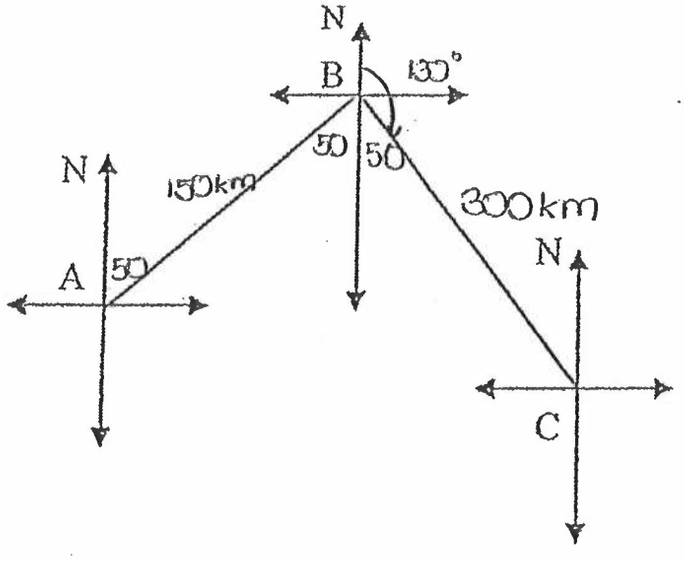
$1 - P(BB)$

$1 - \frac{3}{20}$

$= \frac{17}{20}$ ✓ 1mk for using complement to calculate answer

(15)

A ship sails 150 km from Appleville (A) to Brooktown (B) on a bearing of 050° T.
It then sails on a bearing of 130° T to Cook (C) which is 300 km from Brooktown.



Label the diagram above showing all the essential features **1**

(a) Find the distance from Appleville (A) to Cook (C) **2**
(Give your answer correct to two decimal places)

$\angle ABC = 50 + 50 = 100°$

$AC^2 = 150^2 + 300^2 - 2(300)(150)\cos 100°$ ✓ 1mk for correct use of cosine rule

$b = 375.95$ (2 d.p)

357.95 ✓ 1mk for correct solution

(b) What is the bearing of Appleville (A) from Cook (C)? **2**
(Give your answer to the nearest minute)

$$\frac{\sin \angle BCA}{50} = \frac{\sin 100}{357.95}$$

$$\sin \angle BCA = \frac{150 \sin 100}{357.95}$$

$\angle BCA = 24°22'$ ✓ 1mk for $\angle BCA$

Bearing $= 360 - (50 + 24°22')$
$= 285°38'$ ✓ 1mk for bearing

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Question 16

For the curve $y=\frac{x^3}{3}-3x^2+8x+5$

(a) Find the coordinates of the stationary points and distinguish their nature. **3**

$y' = x^2 - 6x + 8$

for stationary points let $y'=0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4)=0$

$x = 2$ or 4 ✓ finding x co-ords of stat points.

x	0	2	3
y'	8	0	−1

x	3	4	5
y'	−1	0	3

✓ co-ordinates of both stationary points

$\therefore \left(2, \frac{35}{3}\right)$ is a maximum turning point

$\left(4, \frac{31}{3}\right)$ is a minimum turning point ✓ determining their nature.

(b) Find the point of inflection. **2**

$y'' = 2x - 6$

for possible P.O.I let $y''=0$

$2x - 6 = 0$

$2x = 6$

$x = 3$

test

x	0	3	4
y''	−6	0	2

✓ showing an appropriate test.

since y'' changes sign

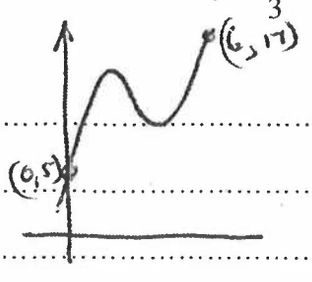
∴ concavity changes

$\therefore (3, 11)$ is a point of inflection ✓ point of inflection

(c) What is the maximum value of $y=\frac{x^3}{3}-3x^2+8x+5$ for the domain $x \in R\,[0,6]$? **2**

$f(0) = 5$

$f(6) = 17$



The maximum value of the function in the given domain is 17 ✓✓

(only 1 mark for (6, 17))

Question 17

(a) Evaluate $\int_2^3 \frac{x}{x^2-1}\,dx$. (Write your answer as an exact value) **2**

$= \frac{1}{2}\int_2^3 \frac{2x}{x^2-1}\,dx$

$= \frac{1}{2}\left[\ln|x^2-1|\right]_2^3$ ✓ for integrating correctly

$= \frac{1}{2}(\ln 8 - \ln 3)$ ✓

$= \frac{1}{2}\ln\frac{8}{3}$ (or equivalent eg $\ln\sqrt{\frac{8}{3}}$)

(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$. (Write your answer as an exact value) **3**

$= \left[\ln|\sin x|\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓ for integrating correctly

$= \ln\left|\sin\frac{\pi}{2}\right| - \ln\left|\sin\frac{\pi}{6}\right|$

$= \ln 1 - \ln\frac{1}{2}$ ✓

$= -\ln\frac{1}{2}$ ✓ $= \ln\left(\frac{1}{2}\right)^{-1} = \ln 2$

Question 18

Differentiate $\frac{x}{\tan 2x}$. **2**

let $u = x \quad v = \tan 2x$

$u' = 1 \quad v' = 2\sec^2 2x$ ✓ differentiating correctly

$y' = \frac{\tan 2x - 2x\sec^2 2x}{\tan^2 2x}$ ✓ correctly using quotient rule.

Question 19

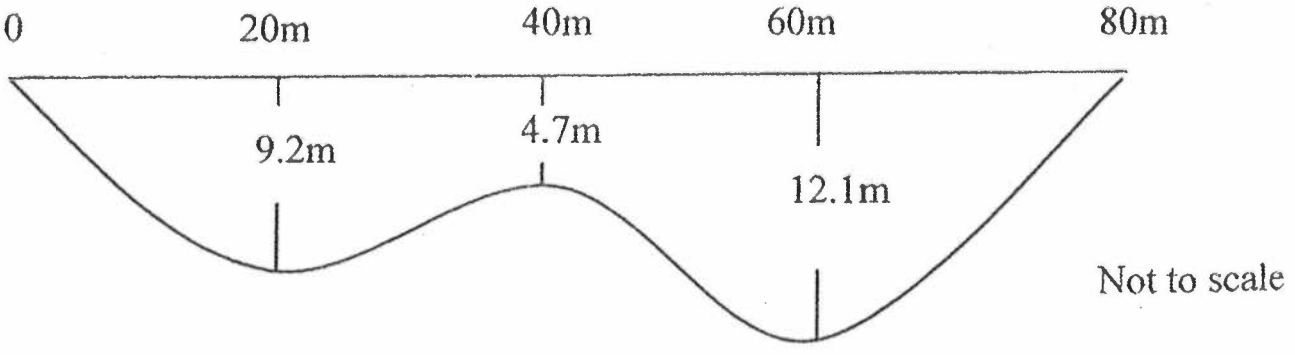
Expand and simplify $(\operatorname{cosec} A+1)(\operatorname{cosec} A-1)$. 2

$= \operatorname{cosec}^2 A - 1$ ✓ expanding correctly

$= \cot^2 A$ ✓ simplifying

Question 20

The following diagram represents a cross-section through a river.
The depth of the river is marked every 20 metres.



Use the trapezoidal rule with 5 function values to estimate the area of the cross-section. 2

$A \doteqdot \frac{h}{2}\left[f(a) + f(b) + 2\left(f(x_1) + f(x_2) + f(x_3)\right)\right]$

$\doteqdot \frac{20}{2}\left[0 + 0 + 2(9.2 + 4.7 + 12.1)\right]$ ✓

$\doteqdot \boxed{520 \text{ m}^2}$ ✓

note h = width of each strip

$A = \frac{b-a}{2n}\left[f(a) + f(b) + 2\left(f(x_1) + f(x_2) + f(x_3)\right)\right]$

n is the number of sub intervals (ie no. of strips)

Question 21

(a) The temperature in the kitchen of Nameeta's new home is controlled by a thermostat.

The temperature can be modelled by the equation: $T = 21.7 + \sin\left(\frac{t}{2}\right)$ where

T is the temperature in degrees Celcius and t is the time in minutes since Nameeta entered the kitchen.

How long will Nameeta have to wait before the temperature gets to 22.5° C **2**

$22.5 = 21.7 + \sin\left(\frac{t}{2}\right)$

$0.8 = \sin\frac{t}{2}$ ✓

$\frac{t}{2} = 0.92729...$ many forgot to use RADIANS !!

$t = 1.8545...$ minutes ✓

(1 min 51 seconds)

(b) The volume V cm^3 of a balloon is increasing such that the volume at any time t seconds is given by $V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$.

Find the rate at which the volume is increasing when $t = 2$ **2**

(write your answer in exact form)

$\frac{dV}{dt} = \pi t^2 - \frac{\pi t}{3}$ ✓ finding $\frac{dV}{dt}$

when $t = 2$

$\frac{dV}{dt} = \pi \times 2^2 - \frac{\pi \times 2}{3}$

$= 4\pi - \frac{2\pi}{3}$ ✓

$= \boxed{\frac{10\pi}{3}}$ cm^3 s^{-1}

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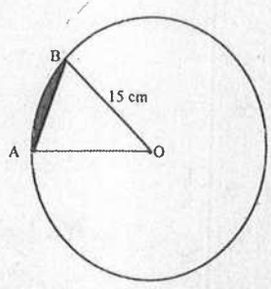
Question 22



Markers Comments

The diagram is a circle centre O and radius 15 cm. The triangle AOB is an equilateral triangle.

AB is a chord of the circle.

* Some failed to see ↑

(a) State in radians the size of angle AOB. **1**

$\angle AOB = \frac{\pi}{3}$ ✓

* Asked for RADIANS not degrees

(b) Calculate the area of the shaded region. (Write your answer as an exact value) **3**

= area of sector − area of triangle

$= \frac{1}{2} r^2 \theta - \frac{1}{2} ab \sin C$

* Learn use formula correctly 'Radians' vs Degrees

$= \frac{1}{2} \times 15^2 \times \frac{\pi}{3} - \frac{1}{2} \times 15^2 \times \sin\frac{\pi}{3}$ ✓

$= \frac{225\pi}{6} - \frac{1}{2} \times 15^2 \times \frac{\sqrt{3}}{2}$ ✓

* Well done by most.

* Some didn't answer exact (−1) mk

$= \frac{75\pi}{2} - \frac{225\sqrt{3}}{4}$

OR $\frac{150\pi - 225\sqrt{3}}{4}$ ✓

OR $\frac{225}{2}\left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$

Anything that was 'exact'

Question 23

A uniform continuous random variable with probability density function is shown below.

$$f(x) = \begin{cases} ax^2, & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of a. **2**

$\int_0^6 ax^2\,dx = 1$

$\checkmark \left[\frac{ax^3}{3}\right]_0^6 = 1$

$\frac{216a}{3} - 0 = 1$

$216a = 3$

$a = \frac{3}{216}$

$a = \frac{1}{72}$ ✓

* well done by most

(b) Find $P(3 \le X \le 5)$ **3**

$\int_3^5 \frac{x^2}{72}\,dx$

$= \frac{1}{72}\left[\frac{x^3}{3}\right]_3^5$ ✓

$= \frac{1}{72} \times \left[\frac{125}{3} - \frac{27}{3}\right]$ ✓

$= \frac{49}{108}$ ✓

* some 'carry on' marks awarded

fraction + decimal + % accepted

(0.4537... also accepted)

Question 24

The circle $x^2 + 6x + y^2 - 4y - 3 = 0$ is reflected in the x-axis. **3**

Sketch **on the next page** the **reflected** circle, showing the coordinates of the centre and the radius.

$x^2 + 6x + y^2 - 4y = 3$

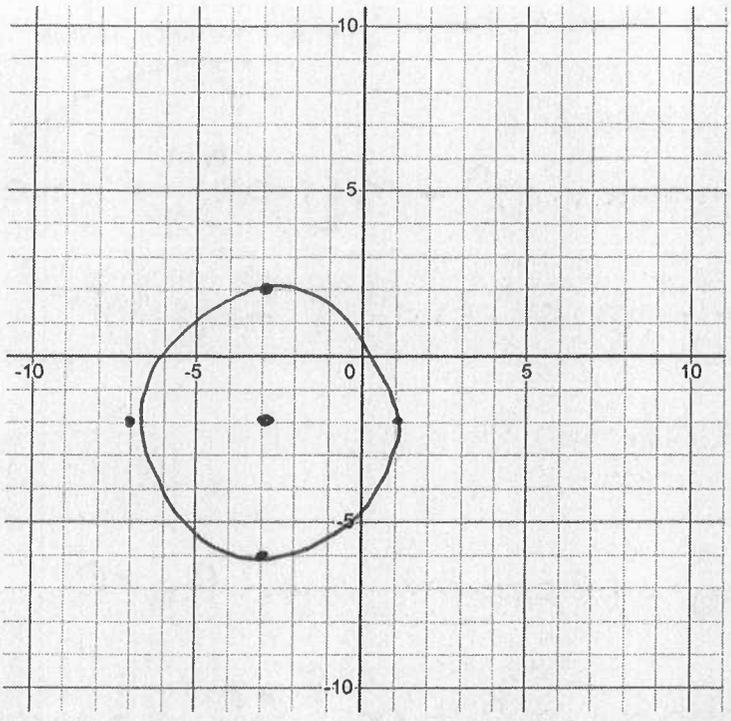
$(x+3)^2 + (y-2)^2 = 3 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 16$

centre $(-3, 2)$ ✓

radius $= 4$ ✓

Reflection in x axis centre $(-3, -2)$



✓ reflection sketch

* had to cross relevant axis

* some v. poor sketches

* many incorrectly reflected in y axis!

Question 25

Jason deposits an amount of \$100 000 into an account which pays compound interest of 2% per annum, added to the account at the end of each year.
Immediately after the interest is added, Jason makes a withdrawal for expenses for the coming year.
The first withdrawal is \$M. Each subsequent withdrawal is 10% greater than the previous one.

Let $\$A_n$ be the amount of money in the account after the nth withdrawal.

(a) Show that $A_2 = 100\,000(1.02)^2 - M(1.02+1.10)$ **1**

$A_1 = 100\,000 \times 1.02 - m$ ✓ had to have this.

$A_2 = [100\,000 \times 1.02 - m] \times 1.02 - m \times 1.10$ } & develop

$= 100\,000 \times 1.02^2 - m \times 1.02 - m \times 1.10$ }

$= 100\,000 \times 1.02^2 - m[1.02 + 1.1]$ (QED)

(b) Show that $A_3 = 100\,000(1.02)^3 - M[(1.02)^2 + (1.02)(1.10) + (1.10)^2]$ **1**

$A_2 = 100\,000 \times 1.02 - m(1.02 + 1.1)$ from part a)

$A_3 = [100000 \times 1.02 - m(1.02 + 1.1)] \times 1.02 - m \times 1.1^2$

something like highlighted ✓

$= 100\,000 \times 1.02^2 - m[1.02^2 + 1.1(1.02)] - m \times 1.1^2$

$= 100\,000 \times 1.02^2 - m[1.02^2 + 1.1(1.02) + 1.1^2]$

* some wrote out part (a) solution then expanded the A_3 given.

* Students need to DEVELOP the series not fudge it!

* Difficult!! * this term here was the error for many.

(c) Hence, write an expression for A_n 1

$A_n = 100\,000 \times 1.02^n - m\left[1.02^{n-1} + (1.02)^{n-2}(1.1) + \ldots\right.$

$\left. \ldots (1.02)(1.1)^{n-2} + (1.1)^{n-1}\right]$

(d) Show there will be NO money in the account when $M = \dfrac{8000}{\left[\left(\dfrac{1.10}{1.02}\right)^n - 1\right]}$ 2

No money in account when $A_n = 0$

$0 = 100\,000 \times 1.02^n - m\left[\text{GP } a = 1.02^{n-1},\ r = \dfrac{1.10}{1.02},\ n = n\right]$

$0 = 100\,000 \times 1.02^n - m\left[\dfrac{1.02^{n-1}\left[\left(\dfrac{1.10}{1.02}\right)^n - 1\right]}{\dfrac{1.10}{1.02} - 1}\right]$

$m\left[\dfrac{1.02^{n-1}\left[\dfrac{1.10^n}{1.02^n} - 1\right]}{\dfrac{1.10 - 1.02}{1.02}}\right] = 100\,000 \times 1.02^n$

$m\left[\dfrac{1.02^{n-1}\left[\dfrac{1.10^n}{1.02^n} - 1\right]}{\dfrac{0.08}{1.02}}\right] = 100\,000 \times 1.02^n$

$m\left[1.02^{n-1}\left[\dfrac{1.10^n}{1.02^n} - 1\right]\right] \times \dfrac{1.02}{0.08} = 100\,000 \times 1.02^n$

$$m\left[1.02^n\left[\frac{1.10^n}{1.02^n}-1\right]\right] = 8000 \times 1.02^n$$

$$m = \frac{8000 \times 1.02^n}{1.02^n\left[\left[\frac{1.10}{1.02}\right]^n - 1\right]}$$

$$m = \left\{\frac{8000}{\left[\frac{1.10}{1.02}\right]^n - 1}\right\}$$

* Very tricky Question
* 1 mark awarded for adequate progress
* No marks for 'fudging'
* needed to show a series sum where $r = \frac{1.10}{1.02}$ for 1 mark here

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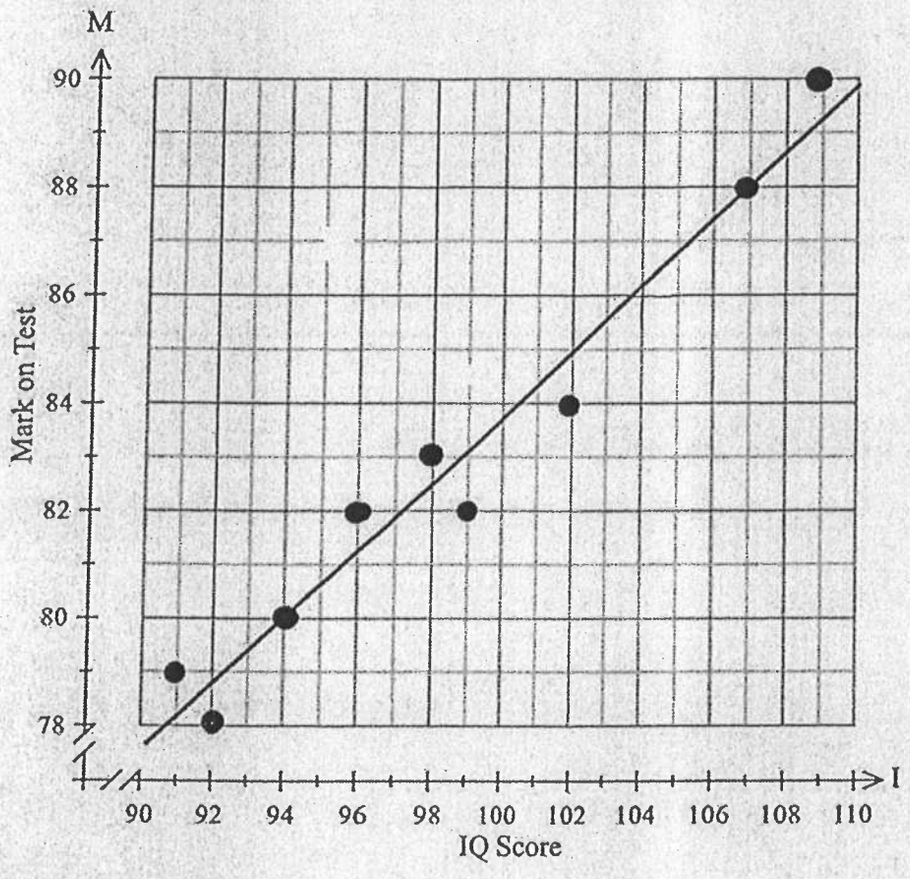
Question 26

The bivariate data in the scatterplot below compare the recorded IQ scores of students with their mark on a class test out of 100. A line of best fit was also drawn.



(Note: Both axes are truncated)

(a) Calculate the gradient of the line of best fit **1**

$m = \frac{88-80}{107-94} = \frac{8}{13} = 0.61538\ldots$

Accepted $\frac{5}{8}$ or any answer close to 0.6

(b) Describe the correlation between the IQ score and the mark on the test in terms of **direction** and **strength**. **2**

Strong Positive.

Question 27

A continuous probability distribution is given by $f(x)=\frac{3x^2}{279}$ defined in the domain $[4,7]$

(a) Find the cumulative distribution function. 3

$$\int_4^x \frac{3x^2}{279}\,dx = \frac{3}{279}\int_4^x x^2\,dx$$

$$= \frac{3}{279}\times\left[\frac{x^3}{3}\right]_4^x = \left[\frac{x^3}{279}\right]_4^x$$

$$\therefore = \frac{1}{279}(x^3-4^3) = \frac{x^3-64}{279}$$

(b) Find the median of the continuous probability distribution. 2

For median CPD $= 0.5$

$$\therefore \frac{x^3-64}{279} = 0.5$$

$$x^3 = 0.5\times 279 + 64 = 203.5$$

$$x = 5.88\ldots$$

Question 28

Evaluate $\int_1^3 x^2(x^3+8)^2\,dx$ 2

By formula $\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1}+c$

$$= \frac{1}{3}\int_1^3 3x^2(x^3+8)^2\,dx$$

$$= \left[\frac{1}{3}\times\frac{(x^3+8)^3}{8}\right]_1^3 = \frac{1}{9}(42146)$$

OR $\frac{d}{dx}(x^3+8)^3 = 3(x^3+8)^2\cdot 3x^2$

$$= 9x^2(x^3+8)^2$$

$$\therefore \frac{1}{9}\int_1^3 \frac{d}{dx}(x^3+8)^3\,dx$$

$$= \left[\frac{1}{9}(x^3+8)^3\right]_1^3$$

$$= \frac{1}{9}(42875-729)$$

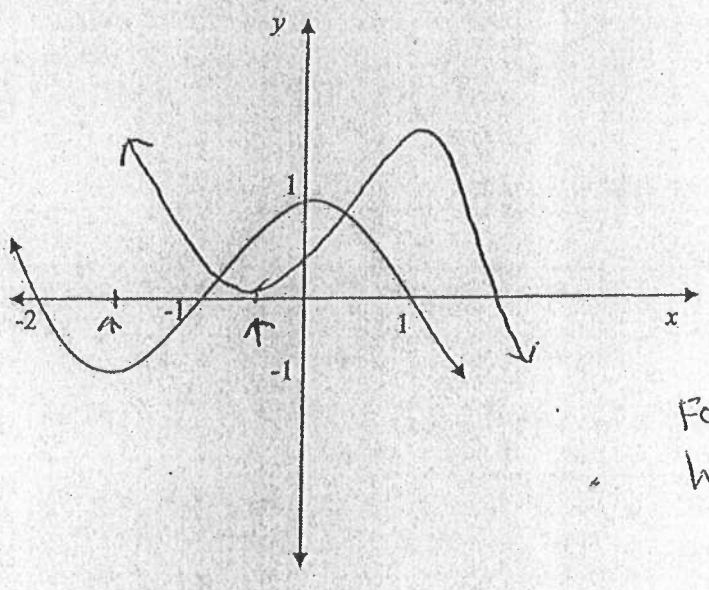
$$= \frac{42146}{9}$$

In decimals (4682.89)

2 dp.

Question 29

(a) The graph of $y = f(x)$ is shown.



Shift Right 1
translate up 1

For full marks 1st t.p. had to be above/on x-axis.

On the graph above draw the curve, $y = f(x-1)+1$, showing all important features. **2**

Question 30

A class of students sat a test worth 30 marks. The marks were normally distributed.

The mean was 18 and the standard deviation was 1.6.

(a) Calculate Julie's z-score if her mark was 22. **1**

$z = \frac{x-\mu}{\sigma} = \frac{22-18}{1.6} = 2.5$

(b) If Dan's z-score was 1.25, calculate his actual mark. **1**

$1.25 = \frac{x-18}{1.6}$

$2 = x-18$

$x = 20$

Question 31

A psychologist proposes that the ability of a child to memorise during the first five years can be modelled by the function $f(x)=1+x\log_e x \quad 0<x\leq 5$

(i.e the ability to memorise at age x years is $f(x)$)

(a) During which month is the ability at a minimum in the first five years 3

$f(x) = 1 + x\ln x$

$f'(x) = \ln x + 1$

for s.p. $f'(x) = 0$

$\ln x + 1 = 0$

$\log_e x = -1$

$e^{-1} = x = 0.367\ldots$ yrs.

for month value $\times 12$

$= 4.41$ months

$\therefore$ 5th month.

$x\ln x$

$u \times v$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$\therefore$

$\frac{d}{dx}(x\ln x) = \ln x + 1$

(b) When is it a maximum during this period. Give a reason for your answer. 2

Maximum occurs when $x = 5$

Graph:

$f(x)$ is a continuous fn.

$f'(x) > 0$ (incr)

Graph: $x = 5$

1 min t.p. $f''(x) = \frac{1}{x}$

$f(\frac{1}{e}) = 2.71 > 0 \therefore$ MIN

Possible reasons:

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Question 32

Evaluate $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y$, given that $y=xe^{-2x}$. **2**

$y = xe^{-2x}$

$y' = -2xe^{-2x} + e^{-2x}$ ← 1mk

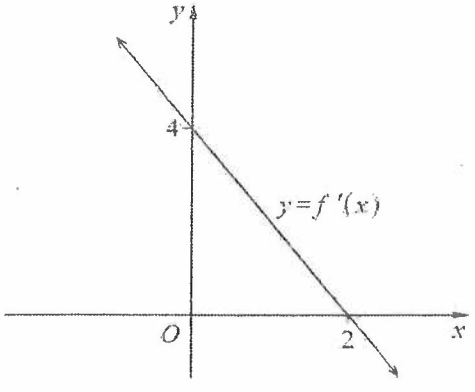
$y'' = -2x \cdot -2e^{-2x} + e^{-2x} \cdot -2 - 2e^{-2x}$

$y'' + 4y' + 4y = 4xe^{-2x} - 4e^{-2x} - 8xe^{-2x} + 4e^{-2x} + 4xe^{-2x}$

$= 0$ ← 1mk

Question 33

The graph of the derivative of a function $y = f'(x)$ is shown below. **3**



$f'(x) = mx + b$

$b = 4 \quad m = -2$

$\therefore f'(x) = -2x + 4$ ← 1mk

$\therefore f(x) = -x^2 + 4x + c$

The curve $y = f(x)$ has a maximum value of 12.

What is the equation of $y = f(x)$?

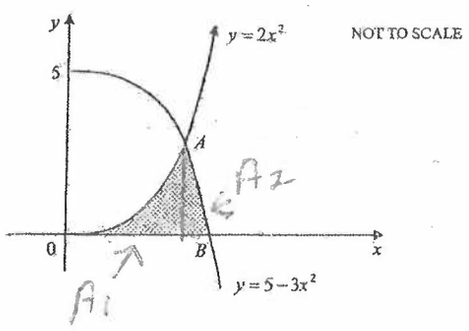
Max occurs when $x = 2 + y = 12$

$12 = -4 + 8 + c$

$\therefore c = 8$ ← 1mk

$\therefore y = -x^2 + 4x + 8$ ← 1mk

Question 34



The shaded region OAB is bounded by the parabolas $y=2x^2$, $y=5-3x^2$ and the x-axis.

Point A is the intersection of the two parabolas and point B is the x-intercept of the parabola $y=5-3x^2$

(a) Show that the x- coordinates of A and B are 1 and $\frac{\sqrt{5}}{\sqrt{3}}$ respectively. **2**

(A) $2x = 5-3x^2$

$5x^2 = 5$

$x = \pm 1$

$\therefore$ A is $x=1$

(B) $5-3x^2 = 0$

$x^2 = 5/3$

$\therefore$ B is $x = \frac{\sqrt{5}}{\sqrt{3}}$

1mk for EACH POINT CLEARLY SHOWN

(b) Find the exact area of the shaded region OAB. **3**

A① $\int_0^1 2x^2\,dx$

$= \left[\frac{2x^3}{3}\right]_0^1$

$= \frac{2}{3}u^2$

A② $\int_1^{\sqrt{5}/\sqrt{3}} (5-3x^2)\,dx$

$= \left[5x - x^3\right]_1^{\sqrt{5}/\sqrt{3}}$

$= \left(\frac{5\sqrt{5}}{\sqrt{3}} - \frac{5\sqrt{5}}{3\sqrt{3}}\right) - (4)$

$= \frac{10}{3}\frac{\sqrt{5}}{\sqrt{3}} - 4$

1mk EACH

$\therefore$ TOTAL $= A_1 + A_2$

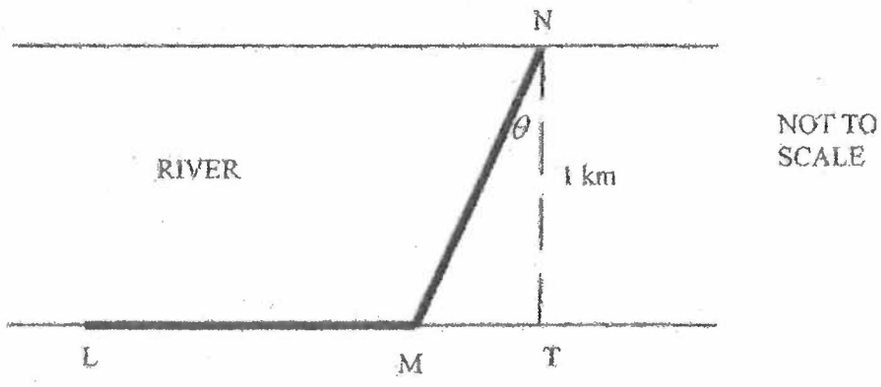
$= \frac{10}{9}\left[\sqrt{15} - 3\right]u^2$ OR EQUIVALENT

1mk for correct answer.

Question 35

It is desired to construct a cable link between two points L and N, which are situated on opposite banks of a river of width 1 km. L lies 3 km upstream from N. It costs 3 times as much to lay a length of cable underwater as it does to lay the same cable overland.

The following diagram is a sketch of cables where θ is the angle where NM makes with the direct route across the river.



Note: $MN = \sec\theta$ and $MT = \tan\theta$

(a) If segment LM costs c dollars per km, prove the total cost (T) of laying the cable is given by $T = 3c - c\tan\theta + 3c\sec\theta$ **2**

$LM = LT - MT$

$LT = 3 \quad MT = \tan\theta$

$LM = 3 - \tan\theta$

Cost of $LM = 3c - c\tan\theta$

$MN = \sec\theta$

Cost of $MN = 3c\sec\theta$

$\therefore T = 3c - c\tan\theta + 3c\sec\theta$

1 mk for EACH segment (LM & MN) CLEARLY EXPLAINED !!!

(b) At what angle should the cable cross the river in order to minimise the total cost of laying it. **3**

$T = 3c - c\tan\theta + 3c\sec\theta$

$= 3c - c\tan\theta + 3c(\cos\theta)^{-1}$

$T' = -c\sec^2\theta - 3c(\cos\theta)^{-2}\cdot -\sin\theta$

$= -c\sec^2\theta + 3c\frac{\sin\theta}{\cos\theta}\cdot\sec\theta$

$= -c\sec^2\theta + 3c\tan\theta\sec\theta$

$= -c\sec^2\theta + 3c\frac{\sin\theta}{\cos^2\theta}$

1 mk for one of these

Let $T' = 0$

$0 = \frac{-c}{\cos^2\theta} + \frac{3c\sin\theta}{\cos^2\theta}$

$0 = -c + 3c\sin\theta$

$\sin\theta = \frac{1}{3}$

$\theta = 19°28'\ (0{-}34^c)$ 1 mk for correct angle

Test: $\theta = 19°$ $T' < 0$ ∪

$\theta = 20°$ $T > 0$

1 mk for correct TEST

$\therefore \theta = 19°28'$ gives minimum

End of Exam